ASTRONOMY 8400 – SPRING 2024
Homework Set 4 – Answers

1.a)

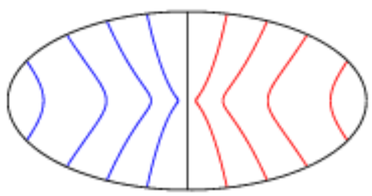
$$M(R) = \sigma \pi R^2 \quad \text{where } \sigma = \text{mass/area} = \text{constant}$$

$$v(R) = \sqrt{\frac{GM}{R}} = \sqrt{\frac{G\sigma\pi R^2}{R}} = \sqrt{G\sigma\pi R}$$

b) $v_r = v(R)\cos\phi \sin i = \sqrt{G\sigma\pi}\sqrt{R}\cos\phi\sin i$

c) $R = k\dfrac{v_r^2}{\cos^2\Phi}$ where k = const.

Isovelocity plot using IDL and $R(\Phi)$ for different vr is given below. Note that the contours are properly spaced for equal intervals of v_r ($R \propto v_r^2$ at Φ=0). (However, I didn't count off if they weren't.)



2. This involves the "radius of influence" equation for a SMBH

a) The resolution of *HST* in the optical is ~0.1", which corresponds to a projected linear distance of 1.50 x 10^{19} cm (= 4.85 pc) at a distance of 10 Mpc. So:

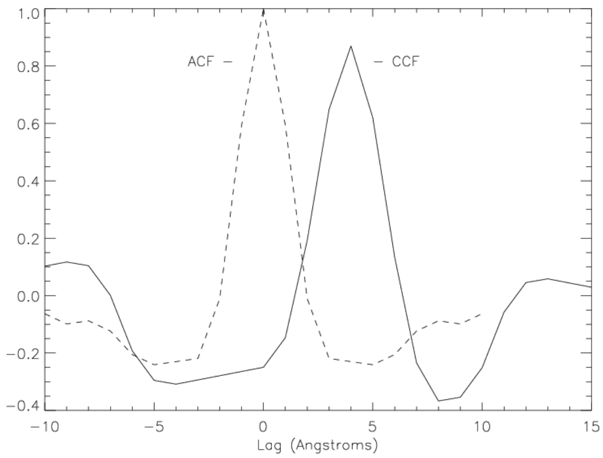
$$M_\bullet > \frac{r\sigma_*^2}{G} = 9.0 \text{ x } 10^{40} \text{g} = 4.5 \text{ x } 10^7 M_\odot$$

b) At 1", the projected linear size at 10 Mpc is 1.50 x 10^{20} cm (= 48.5 pc), and the minimum mass is 4.5 x 10^8 $M_\odot$.

c) From the graph of Kormendy et al., the minimum bulge luminosity in absolute B magnitude is $M_B \approx$ -18 (~10^9 $L_\odot$) for the HST case and $M_B \approx$ -20.5 (~10^{10} $L_\odot$)for the optical case (with large dispersions for both). These correspond to moderate-size ellipticals and bulges (like the Milky Way) for the former and large ellipticals and bulges for the latter.

d) From the Gebhardt et al. correlation, M = 1.5 x 10^8 $M_\odot$. For the case above, you would probably detect the SMBH with HST, but not from the ground.

3. a) b) I calculated the CCF and ACF using the IDL procedure C_CORRELATE. The CCF is somewhat sensitive to the "window" used, which is the length of spectrum on either side. If you add a bunch of 1.0s on either side of the spectra (I added some) it should be symmetric. Here's the CCF for the star/galaxy, and the ACF for the star.



c) CCF peak is at 4Å:

$$v_r = \frac{\Delta\lambda}{\lambda} c = \frac{4}{8542}\left(3.0 \text{ x } 10^5\right) = 140 \text{ km s}^{-1} \text{ for the galaxy.}$$

You get the same answer from the centroids (in this case minima) of the absorption lines.

d) This wasn't a great question, since the profiles are not really Gaussians. The FWHM is 3Å (105 km s^{-1}) for the galaxy and 2Å (70 km s^{-1}) for the star. Assuming they are Gaussians, and the stellar profile gives the line-spread function (LSF), the intrinsic FWHM of the galaxy absorption line is:

$$\text{FWHM}_{\text{intr}} = \sqrt{\text{FWHM}_{\text{obs}}^{\,2} - \text{FWHM}_{\text{LSF}}^{\,2}}$$

$$\text{FWHM} = \sqrt{(105)^2 - (70)^2} = 78 \text{ km s}^{-1}$$

The intrinsic velocity dispersion is

$$\sigma = \text{FWHM}/2.355 = 33 \text{ km s}^{-1}$$

Note that the cross-correlation of two spectra with many features is usually done with the wavelength scale in lnλ. The reason is that for a given radial velocity, $\lambda \propto \Delta\lambda$, so the shift in wavelength is not constant. However Δlnλ is:

$$\Delta \ln \lambda = \frac{\Delta\lambda}{\lambda} = \frac{v_r}{c}$$