Secular & Statistical Parallax

Spring 2024: Cosmic Distance Ladder Part III

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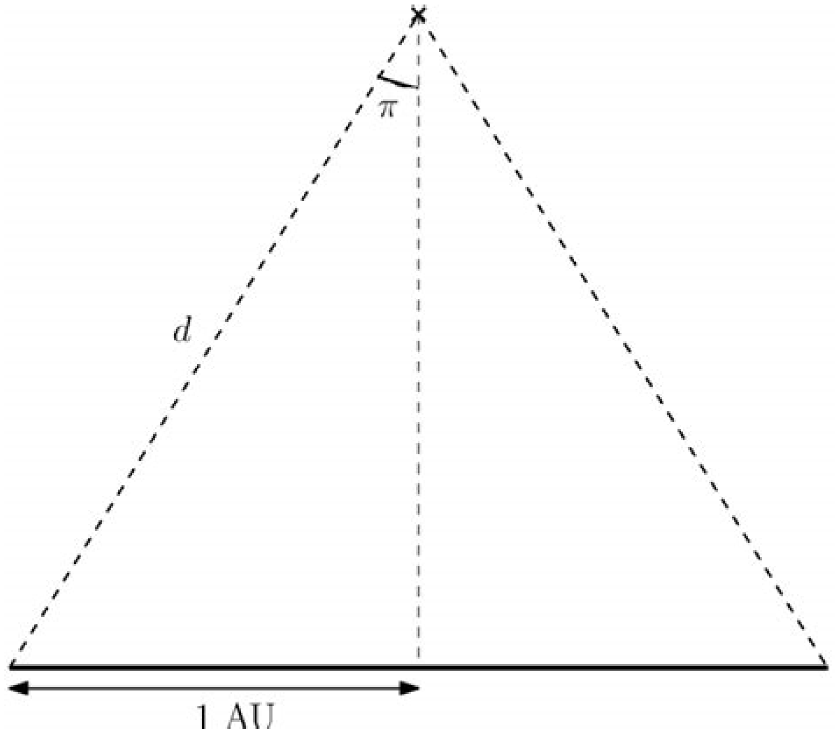
what is secular/statistical parallax?

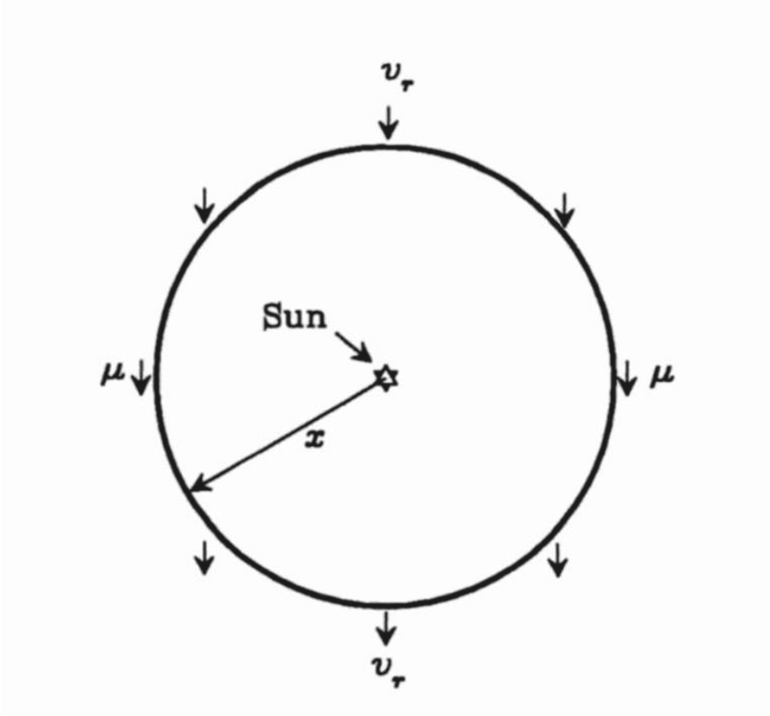
1. use average motions (proper motions, radial velocities) of a large number stars to estimate distance
2. equal luminosity & equal apparent brightness → same distance
3. for enough stars: motions randomly distributed → average motion relative to Sun gives us distance

longer baseline

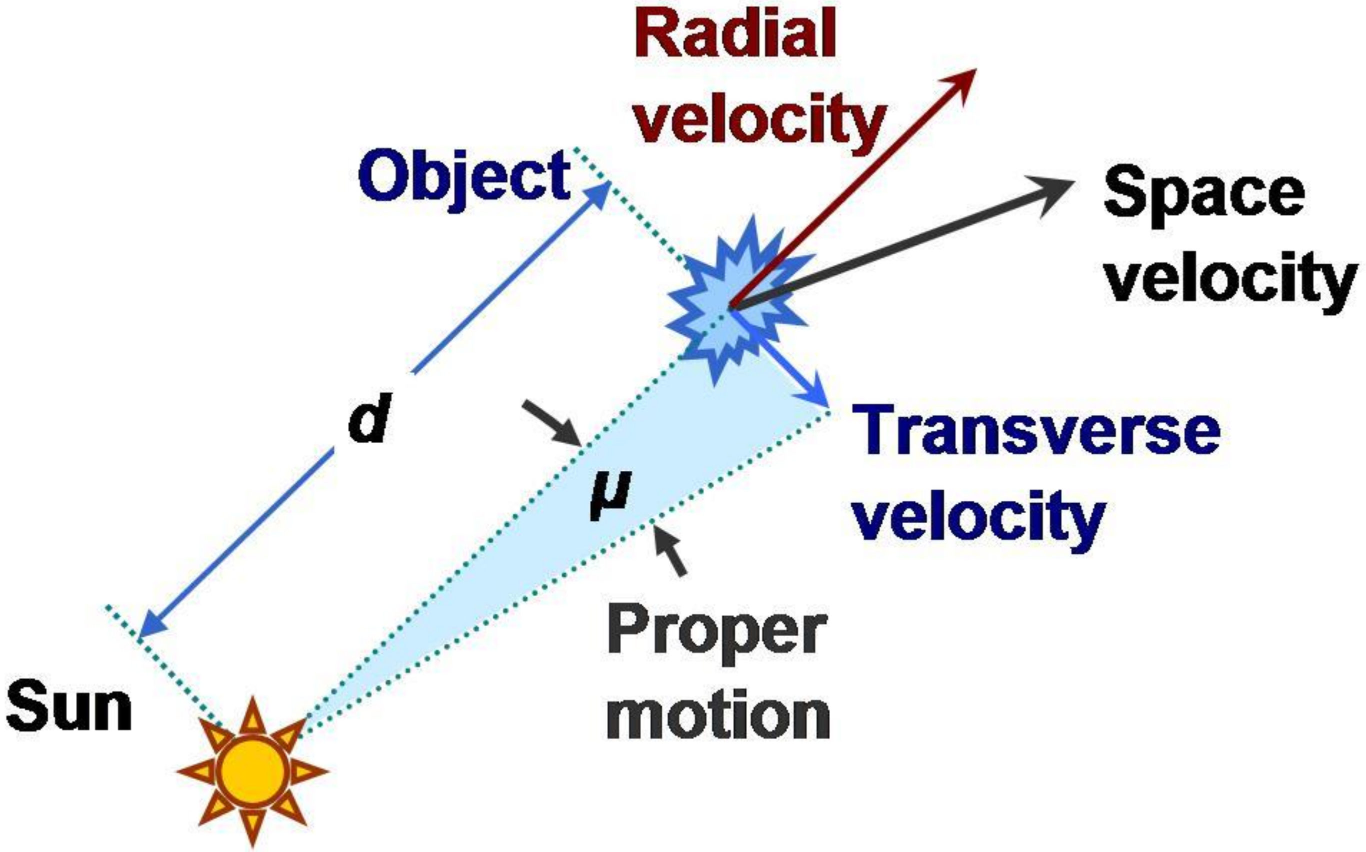
Trigonometric Parallax: 2 au/yr



Secular Parallax: 4 au/yr (disk), 40 au/yr (halo)



stars move through space…



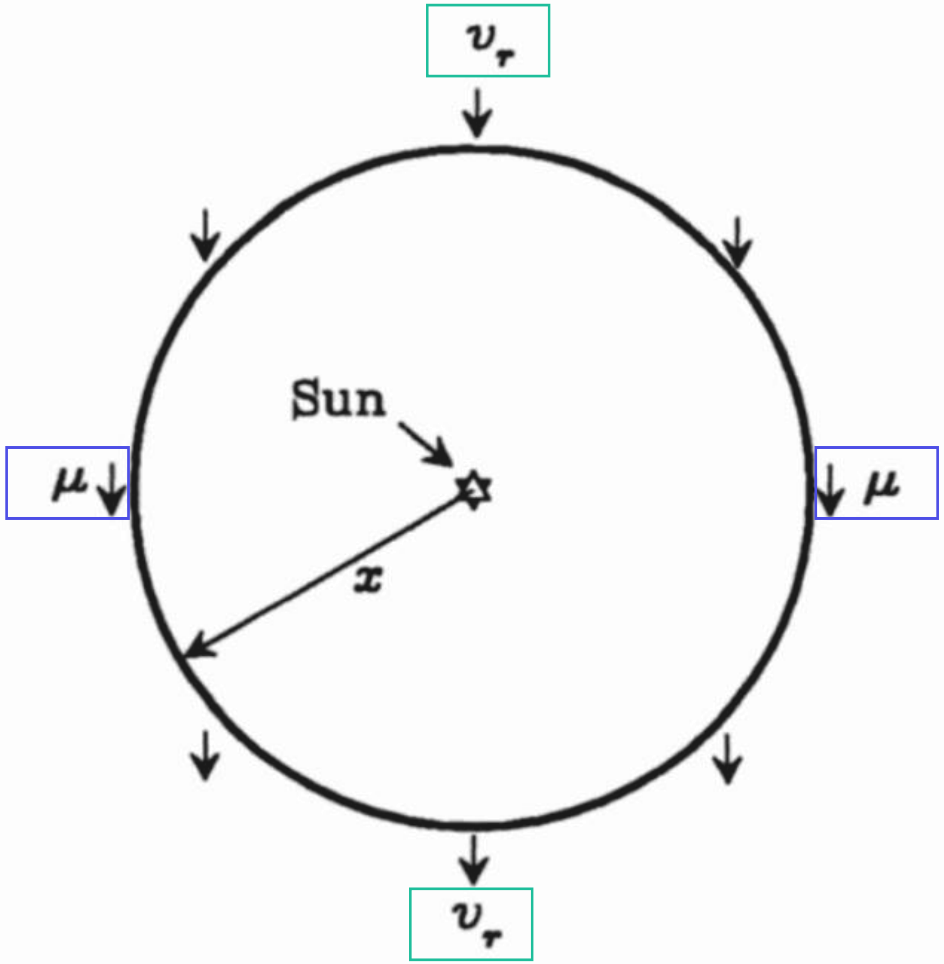
consider shell of stars around Sun

$$x = v_r/\mu_\perp$$

…but relies on assumption that all stars have same motion relative to Sun

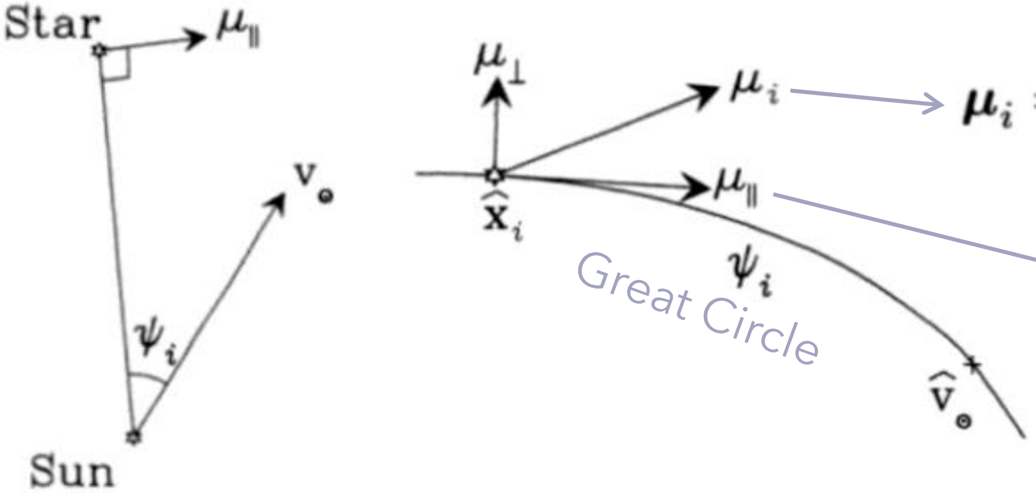
INSTEAD…assume *mean* velocity relative to Sun:

$$\sum_i v_i = 0$$



use heliocentric velocity relative to $v_\odot$:

$$u_i = v_i - v_\odot$$

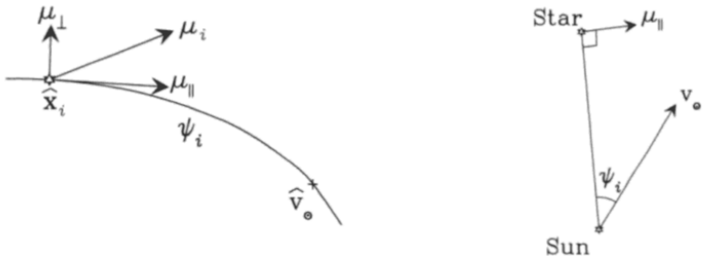


$$\boldsymbol{\mu}_i = \frac{(\mathbf{u}_i \times \hat{\mathbf{x}}_i) \times \hat{\mathbf{x}}_i}{x_i} = \frac{((\mathbf{v}_i - \mathbf{v}_\odot) \times \hat{\mathbf{x}}_i) \times \hat{\mathbf{x}}_i}{x_i}.$$

$$\mu_{\parallel i} = \frac{\boldsymbol{\mu}_i \cdot \hat{\mathbf{v}}_\odot}{\sin \psi_i}$$

For each star at distance x_i: $$x_i = v_\odot \frac{\sin^2 \psi_i}{\mu_{\parallel i} \sin \psi_i} - \frac{(\hat{\mathbf{x}}_i \times \hat{\mathbf{v}}_\odot) \cdot (\hat{\mathbf{x}}_i \times \mathbf{v}_i)}{\mu_{\parallel i} \sin \psi_i}.$$

…average over all stars



$$\left\langle x_i = v_\odot \frac{\sin^2\psi_i}{\mu_{\|i}\sin\psi_i} - \frac{(\hat{\mathbf{x}}_i\times\hat{\mathbf{v}}_\odot)\cdot(\hat{\mathbf{x}}_i\times\mathbf{v}_i)}{\mu_{\|i}\sin\psi_i}.\right\rangle$$

$$\boxed{\sum_i v_i = 0}$$

Recall: $\varpi = 1/x$

Secular Parallax: $$\langle\varpi\rangle = \frac{\langle\mu_{\|i}\sin\psi_i\rangle}{v_\odot\langle\sin^2\psi_i\rangle}.$$

**Use when solar motion dominates: $v_\odot > |\hat{x}\cdot v|$

statistical parallax

- assume v_i distributed isotropically → use radial velocities to get distance

Statistical Parallax:

$$\overline{\varpi} = \frac{\langle|\mu_{\perp i}|\rangle}{\langle \underbrace{u_{ri}}_{\text{radial velocity}} + v_\odot \cos\psi_i\rangle}.$$

Average perpendicular proper motion component

Average corrected RV

**Use when random velocities dominate: $v_\odot < |\hat{x} \cdot v|$

secular/statistical parallax regime

Measures Further than Trigonometric Parallax:

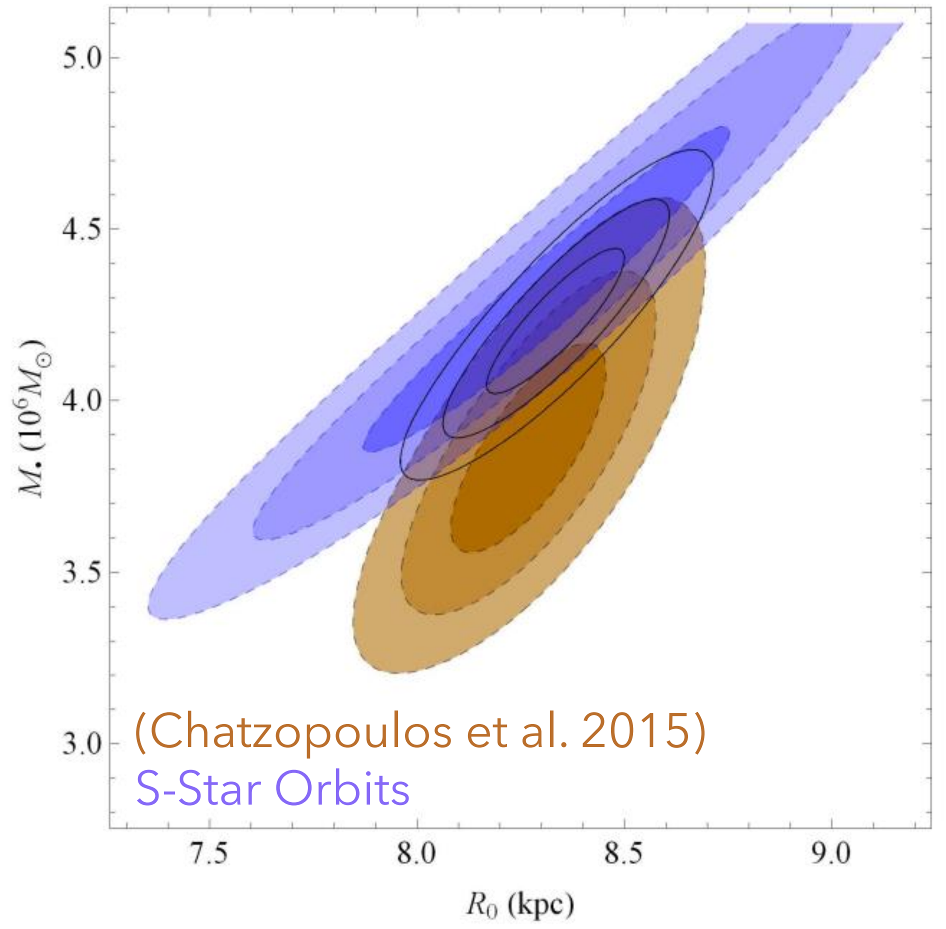
- Distances **~500 pc** (at precision where trigonometric parallax can measure to 200 pc)

- …in recent literature, you can get much further!

application: distance to Galactic Center!

(Chatzopoulos et al. 2015)

- degeneracy: distance to Sgr A* and SMBH mass → need accurate distance measurements to GC
- 2500 RVs from SINFONI IFU spectra, 10000 PMs from AO images
- dynamical model → velocity dispersions
- used statistical parallax to get distance to Galactic nuclear star cluster → $d = 8.33 \pm 0.11$ kpc
- Estimated SMBH mass: $\mathrm{M} = (4.23 \pm 0.14) \times 10^6 M_\odot$



advantages vs. limitations

advantages

- longer baseline than trigonometric parallax → expands to larger distances

limitations

- requires large number of stars of same luminosity/apparent mag
- relies on assumptions about velocity distribution of stars (nod differential Galactic rotation)
- can't reach distances allowed by other techniques
- outdated with current parallax precisions (*Gaia*)

questions?