Astr 8000 – Problem Set # 1 – Due Febuary 5, 2019

1. Suppose an observer is at distance r from the center of a star of radius $r_\star$ which has a uniformly bright surface (i.e., I is independent of μ). Derive analytical expressions for J, H, and K in terms of $\theta_\star = \sin^{-1}(r_\star/r)$, and show that in the limit $r/r_\star \to \infty$, they all have the same dependence on $\theta_\star$ and that $J = H = K$. [25]

2. Show that the relationship between angular diameter α and observed m_1 and model m_2 magnitude is given by

   $$\log \alpha = -0.2(m_1 - m_2 + C)$$

   where C is a constant.

   I have posted on the class Web site a table of computed K-band magnitudes for line-blanketed model atmospheres by Robert Kurucz (http://kurucz.cfa.harvard.edu/grids.html):
   `http://www.astro.gsu.edu/~gies/ASTR8000/rijklp00k2odfnew.dat`

   However, since all magnitude systems are relative to some standards, Kurucz has elected to leave the K magnitudes on an arbitrary scale that needs an additive constant. I want you to make such a calibration in order to derive an expression for predicted angular diameter, α, as as function of stellar effective temperature and apparent magnitude K. Use the relationship above between observed and emitted magnitude, and find the missing constant C needed to use Kurucz' model $K = m_2$ values. Use the set of luminosity class V and IV stars with angular diameter measurements by Hanbury Brown, Davis, & Allen (1974, MNRAS, 167, 121), effective temperatures from Code et al. (1976, ApJ, 203, 417), and observed $K = m_1$ magnitudes from Bouchet et al. (1991, A&AS, 91, 409). For simplicity use the $\log g = 4.5$ models in the Kurucz table. Make a log-log plot of angular diameter (in milli-arcsec) versus effective temperature for $V_0 = 0, 1, \ldots, 10$. The CHARA Array has a resolving power of 0.5 milli-arcsec in the K-band. Estimate the limiting magnitude of resolved stars for temperatures of 5000, 10000, and 30000 K. [25]

   [please turn over]

3. Use the accompanying solar limb-darkening data and absolute continuum specific intensities, $I'_\lambda(0)$ (in units of erg sr^{-1} cm^{-2} s^{-1} Å^{-1}; from page 354 of the 4th edition of Allen's *Astrophysical Quantities*), to carry out the following.

   a. Determine $T(\tau_\lambda)$ from the Eddington-Barbier relation by making a least squares fit to the limb-darkening data (from AQ p. 356) with an expression of the form

$$\frac{I_\lambda(\mu)}{I_\lambda(\mu = 1)} = a_0 + a_1\mu + 2a_2\mu^2.$$

   Tabulate your results at $\tau = 0.2, 0.4, 0.6, 0.8$, and 1.0.

λ(Å)	$\mu = 1.0$	$\mu = 0.8$	$\mu = 0.6$	$\mu = 0.4$	$\mu = 0.2$	$I'_\lambda(0)$
5000	1.00	.877	.744	.599	.425	4.08×10^6
8000	1.00	.924	.843	.744	.615	1.97×10^6
16000	1.00	.959	.906	.838	.744	0.40×10^6

   b. Assume that the ratio of $\kappa_\lambda/\kappa_{5000}$ is independent of depth in the atmosphere so that the 8000 Å and 1.6 μm $T(\tau_\lambda)$ curves can all be transformed to a τ_{5000} optical depth scale. Find the best $\kappa_\lambda/\kappa_{5000}$ values to bring the curves into coincidence, and plot the composite $T(\tau_\lambda)$ curves on a single graph. [25]

4. Calculate the effective temperature of the star β Cas using the data below and posted on the Web site. The star has been resolved by the CHARA Array interferometer (Che et al. 2011, ApJ, 732, 68), and was observed to have an angular diameter of 2.07 mas. Use this value and the integrated flux (ignoring negligible interstellar extinction) to get the effective temperature. (Note that the star is rapidly rotating, so the actual temperature will vary across the surface.) I will post three files giving the flux in the UV, optical, and IR. Tranform these wavelengths and fluxes to Angstroms and units of erg cm^{-2} s^{-1} Å^{-1}. You may find useful the NICMOS conversion tool at
   `http://www.stsci.edu/hst/nicmos/tools/conversion_form.html`
   Plot the fluxes as $\log\lambda$ versus $\log\lambda F_\lambda$, and use this form for linear interpolation over a grid of λ values. Then perform a numerical integration of the flux. Also, if the parallax is $\pi = 59.89 \pm 0.56$ mas, what is the star's radius and radius error (in solar units)? [25]