Astr 8000 – Problem Set #2 – Due February 28, 2019

1. The equation of hydrostatic equilibrium is

$$\frac{dp}{dz} = -\rho g$$

where the pressure derivative with respect to height z in a stellar atmosphere depends on density ρ and gravitational acceleration g. In hot stars, the pressure is the sum of gas pressure p_g and radiation pressure p_R which is given by

$$p_R = (4\pi/c) \int K_\nu \, d\nu$$

where K_ν is the second moment of the intensity field. The depth variation of K_ν can be obtained from the first moment of the transfer equation,

$$\frac{dK_\nu}{dz} = -\chi_\nu H_\nu$$

where H_ν is the Eddington flux (first moment of the intensity field).
(a) Show that by changing the height variable to column mass $dm = -\rho \, dz$, the equation of hydrostatic equilibrium can be written as

$$dp_g/dm = g - (4\pi/c) \int (\chi_\nu/\rho) H_\nu \, d\nu = g(1 - \Gamma)$$

where Γ is the ratio of radiative to gravitational acceleration.
(b) Suppose hot stars contain only fully ionized H and He (abundance Y). If electron scattering alone is taken to provide a lower limit on the opacity, show that

$$\chi_\nu/\rho = n_e\sigma_e/\rho = \sigma_e(1 + 2Y)/(m_H(1 + 4Y))$$

where σ_e is the cross section for electron scattering and m_H is the mass of hydrogen.
(c) Show that

$$\Gamma = \frac{\sigma_e(1 + 2Y)}{m_H(1 + 4Y)} \frac{L}{4\pi cGM}$$

where L is the luminosity, G is the gravitational constant, and M is the mass.
(d) Use the following data to express Γ in terms of $L/L_\odot$ amd $M/M_\odot$:

$\sigma_e = 6.65 \times 10^{-25}$ cm^2 electron^{-1}, $Y = 0.1$, $m_H = 1.673 \times 10^{-24}$ g, $L_\odot = 3.85 \times 10^{33}$ erg s^{-1}, $M_\odot = 1.989 \times 10^{33}$ g, and $G = 6.6726 \times 10^{-8}$ cm^3 g^{-1} s^{-2}.
(e) Use Figure 16 from Köhler et al. (2015, A&A, 573, A71) to estimate a power law version of the mass - luminosity relationship $L/L_\odot = a_0(M/M_\odot)^{a_1}$ appropriate for a $200M_\odot$ ZAMS star. Use this with your expression for Γ to find the maximum possible mass, i.e., where outward radiative equals inward gravitational acceleration. [40]

2. Use the matrix method below to determine the numbers of neutral, once ionized, and twice ionized helium, plus the numbers of neutral and ionized hydrogen, as a function of electron density and temperature. Assume a gas that is composed of H and He with a number ratio of He to H of 0.0794 (solar value). Find the data on ionization potentials and partition functions in Gray, Appendix D. Make a plot of the fractions of each ionic species relative to their total number as a function of temperature from 5000 to 50000 K for $n_e = 10^{14}$ cm^{-3} and for $n_e = 10^{13}$ cm^{-3} (values suitable for main sequence and supergiant stars). Explain the trends you have plotted. [30]

$$\begin{pmatrix} 1 & -N_J/N_{J+1} & 0 & 0 & 0 \\ 0 & 1 & -N_{J+1}/N_{J+2} & 0 & 0 \\ 1 & 1 & 1 & -y & -y \\ 0 & 0 & 0 & 1 & -N_J/N_{J+1} \\ 0 & 1 & 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} n(\mathrm{He}^0) \\ n(\mathrm{He}^+) \\ n(\mathrm{He}^{++}) \\ n(\mathrm{H}^0) \\ n(\mathrm{H}^+) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ n_e \end{pmatrix}$$

3. Consider the extinction and limb darkening properties of main sequence stars of 4000 K, 8000 K, and 16000 K using the web tool *ChromaStar* written by Ian Short:
`http://www.ap.smu.ca/~ishort/OpenStars/ChromaStar/ChromaStar.html`
In the Input panel, set $T_{\rm eff}$, $\log g = 4.4$, and leave the default values for the rest.
(a) In the Output panel, set the narrow band filter to 500 nm and toggle on the switch for an atmosphere plot. Click on the model button. Look for the $\log_{10}$ Extinction plot at the botton of the window, and make a screen copy of the plot region for insertion into a document. Print out a page with these three plots, mark each by $T_{\rm eff}$, and annotate the plots with the important continuum opacity sources.
(b) In the Output panel toggle on the limb darkening coeffs for a detailed print-out, and click on the model button. Copy the two column table below the graphs into files for each temperature. Then make a $(\log \lambda, LDC)$ plot for each temperature, where LDC is the linear limb darkening coefficient.
(c) Compare the plots from parts (a) and (b) and explain the wavelength trends of the limb darkening. Next estimate the extinction coefficient κ at 500 nm and $\tau = 1$, and compare these to the LDC at 500 nm for each temperature. Use these measurements to explain the temperature trend in LDC. [30]