Astr 8000 – Problem Set #4
Due Thursday, April 25, 2019

1. Tabulate and plot the Hβ profile for collisional broadening using the nearest neighbor theory. Underhill & Waddell (1959) express the profile as a sum of Stark components:

$$S(\alpha) = \frac{1}{n^2 f} \sum_k \frac{F_k}{C_k} W\Big(\frac{\alpha}{C_k}\Big)$$

where $n = 2$ for Hβ, $f = 0.11932125$, W is the probability function from nearest neighbor theory (Mihalas eq. 9-87), and the sum is taken over 7 terms (below). Write a program to calculate $S(\alpha)$, and then compare your results with the profile from the Model Microfield Method of Stehlé & Hutcheon (1999, A&AS, 140, 93). I have placed a copy of their tabular data at `http://www.astro.gsu.edu/~gies/Stehle_ba_ba04_profil11.dat`. Find the appropriate block for $T_{\rm eff} = 20,000$ K and $N_e = 10^{15}$ cm^{-3}. Column 1 lists α and column 2 gives $S(\alpha)$ (including Doppler broadening). Plot the results as: (1) $\log S(\alpha)$ versus $\log \alpha$ (like Mihalas Fig. 9-2), and (2) $I(\triangle\lambda)$ versus $\triangle\lambda$. [40]

k	C_k	F_k
1	0.00906869	0.01455833
2	0.01813738	0.08677904
3	0.02720607	0.06365699
4	0.03627476	0.03653854
5	0.04534345	0.03549186
6	0.05441214	0.00152244
7	0.06348083	0.00009515

2. Calculate residual absorption profiles using the Milne - Eddington model

$$R_\nu = (a\sqrt{3} + \frac{b}{1+\beta_\nu})/(a\sqrt{3}+b).$$

Adopt a simple linear source function, $S_\nu = B_\nu = \frac{3}{4}F_\nu(\tau + \frac{2}{3})$, to determine the constants a and b. Take the true absorption profile to be $\beta_\nu = \beta_\circ H(a,v)$ where $H(a,v)$ is the Voigt function (use the IDL library routine VOIGT). Use $a = 10^{-3}$ and $\beta_\circ = 0.1$, 1, 10, 100, 1000, and 10000. Plot the results for $v = 0$ to 10. Measure the equivalent width of these profiles and plot a curve of growth ($\log W$ versus $\log \beta_\circ$). [30]

3. Consider the curve of growth method to determine abundance (Gray Chap. 16). Use the equivalent width measurements for Fe I in the spectrum of α Cen A (Furenlid & Meylan 1990, ApJ, 350, 827; see `http://www.astro.gsu.edu/~gies/ASTR8000/fmfe.dat`), and convert the excitation energy χ in the last column from cm^{-1} to eV. Then use the reference curve of growth in Gray Table 16.2 (copied below) to determine the abundance of iron in α Cen A. Use the graphical technique described on page 398 of Gray, i.e., for each measured line determine $y = \log(W_\lambda/\lambda)$ and $x = \log gf + \log \lambda/\lambda_0 - \log \kappa/\kappa_0 - \theta\chi$ and then find the best fit shift in x to match the reference curve of growth to the full set of measurements. Notes: Assume an effective temperature $T_{\rm eff} = 5710$ K, $\theta = 5040/T_{\rm eff}$, and a microturbulent velocity of $\xi = 0$ km s^{-1}, and use Fig. 8.5(a) (copied below) to make a linear fit of the variations in continuous opacity κ/κ_0 with wavelength in the expression for x. [30]

Table 16.2. *Solar-model curves of growth for λ4500.*

(log W/λ for Fe I, $\chi = 0$ eV, $\log g_n f = 0$, ξ in km/s at the head of the columns.)

log A	0	1	2	3	5
−12.0	−6.66	−6.66	−6.66	−6.66	−6.66
−11.5	−6.17	−6.17	−6.17	−6.16	−6.15
−11.0	−5.71	−5.70	−5.68	−5.68	−5.67
−10.5	−5.33	−5.30	−5.25	−5.23	−5.20
−10.0	−5.07	−5.01	−4.92	−4.86	−4.79
−9.5	−4.93	−4.84	−4.73	−4.62	−4.50
−9.0	−4.82	−4.74	−4.60	−4.50	−4.34
−8.5	−4.71	−4.64	−4.52	−4.41	−4.24
−8.0	−4.58	−4.52	−4.42	−4.32	−4.17
−7.5	−4.39	−4.36	−4.30	−4.22	−4.09
−7.0	−4.17	−4.16	−4.12	−4.09	−3.99
−6.5	−3.93	−3.92	−3.91	−3.90	−3.85
−6.0	−3.69	−3.68	−3.68	−3.67	−3.65
−5.5	−3.44	−3.44	−3.44	−3.43	−3.42